TUXFORD ACADEMY

Further Maths:

GCSE to A-level transition booklet

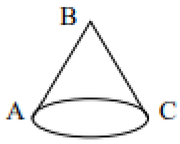
Some of the content covered in Further Maths A-level is *very* different from anything you will have encountered before, both at GCSE and in your Maths A-level.

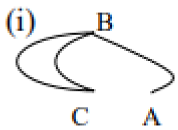
<u>Part 1: Matrices</u>

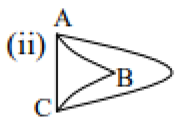
During your taster session you did some work on matrices. Here are some questions for you to have a go at.

1. This diagram shows a map of the roads linking 3 towns A, B and C. The corresponding 'direct route' matrix is shown beside it.

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix} \end{array}$$







For each of the following diagrams construct the direct route matrix.

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{cc} M & T \end{array} \\ \begin{bmatrix} 4 & 6 \\ 3 & 5 \\ 7 & 2 \end{bmatrix} \end{array}$$

2. A café sells 3 main meals A, B, and C each day. On two days the sales of each type are shown in the matrix. If meal A costs £4, meal B costs £5 and meal C costs £3 construct a matrix showing the amount taken for each of the meals on each of the two days. Hence state the total amount taken for each meal over the two days.

$$\mathbf{A} = \begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 2 & 7 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 2 & 3 & -4 \\ -1 & 2 & 5 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} -1 & -4 & 2 \\ -3 & 5 & 6 \end{pmatrix}$$

3. Calculate, if possible,
   (i) **A** + 2**B** (ii) **C** – **D** (iii) 3**A** – 2**C** (iv) 3**D** – **C**

4. Calculate, if possible, the following
   (i) **AB** (ii) **AC** (iii) **BC** (iv) **BD**

If $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ x & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 6 & 2 \\ 4 & y \end{pmatrix}$ find the values of x and y given that $\mathbf{AB} = \mathbf{BA}$.

$\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$.

Find $\mathbf{M}^2 - \mathbf{N}^2$ and $(\mathbf{M}+\mathbf{N})(\mathbf{M}-\mathbf{N})$ and explain why your results are not equal.

$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -3 & 4 \end{pmatrix}$ $\quad \mathbf{B} = \begin{pmatrix} -1 & 3 & 2 \\ 5 & 1 & -2 \end{pmatrix}$ $\quad \mathbf{C} = \begin{pmatrix} 3 & -1 \\ 1 & 2 \end{pmatrix}$ $\quad \mathbf{D} = \begin{pmatrix} 4 & -1 \\ 2 & 5 \\ -3 & 1 \end{pmatrix}$

5.

6.

Part 2: Complex Numbers

Complex numbers are another topic you will cover in Year 12.

1. Your next task is to do some research on what complex numbers are. *You shouldn't spend longer than 45 minutes on this part*.

A website you may want to use and will use during Y12/13 is integral. The following log in details give you access to the website and this runs through examples. (It also has videos on matrices and other topics).

https://my.integralmaths.org/course/view.php?id=84&sectionid=1757

STUDENT ACCOUNT
Username: amst-Tuxford1297
Password: DerivativeDivergence808%

NOTES:

$$\sqrt{-1} = i$$
$$i^2 = -1$$

Numbers of the form ai, where a is a real number, are called *imaginary numbers*, for example $-5i$ or $\sqrt{3}i$

Numbers of the form $a+bi$, where a and b are both real numbers, are called *complex numbers*, for example $2-3i$ or $-0.5+3\sqrt{2}i$

Complex numbers are often given the symbol z

If $z=a+bi$ then its *conjugate* is given the symbol z^* and is defined by $z^*=a-bi$

z multiplied by z^* is always a real number: $(a+bi)(a-bi)=a^2-(bi)^2=a^2+b^2$

If two complex numbers are equal then the real parts are equal to each other, and also the imaginary parts are equal to each other. We can form two separate equations by *equating real and imaginary parts*

If a quadratic equation has real coefficients and $b^2-4ac<0$ then its two roots are complex numbers, which are always conjugates of each other.

2. In the following exercise, $z_1 = 2 - 3i$ and $z_2 = -1 + 5i$. Match each expression on the left with one on the right, and write your answers in the table below

A	$z_1 + z_2$	**1**	$1+5i$
B	i^7	**2**	26
C	$2z_1^*$	**3**	4
D	$\frac{1}{i^3}$	**4**	$-17-7i$
E	$z_2 - z_1$	**5**	$-i$
F	i^{14}	**6**	13
G	$z_2 z_2^*$	**7**	1
H	$\frac{4}{i^8}$	**8**	$-3+8i$
I	$7z_1 + 2z_2$	**9**	$-5+4i$
J	$z_1^* z_1$	**10**	$8i$
K	$z_1^* + z_2^*$	**11**	$5+5i$
L	$3z_2 - 4z_1$	**12**	$1-2i$
M	$5i^2 - 4i^3$	**13**	$-5-i$
N	$\sqrt{-64}$	**14**	$4+6i$
O	iz_2	**15**	-1
P	$i^2 z_2^*$	**16**	i
Q	$\sqrt{25} + \sqrt{-25}$	**17**	-7
R	$\frac{1}{i^{200}}$	**18**	$1+2i$
S	$i\sqrt{-49}$	**19**	$12-11i$
T	$z_1 z_2^*$	**20**	$-11+27i$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T

Part 3: Discrete mathematics – Linear Programming

Part of further mathematics is applied. You will dig deeper into mechanics and you will also learn about discrete mathematics. This is more about making decisions and logic thinking.

1. For this part, you should do some research on linear programming. The website given on Part 2 may be of use again, but there are lots of websites and youtube videos. *You shouldn't spend longer than 30 minutes on this part*.

2. Have a go at the following question.
   A teacher is planning a trip for 174 students and 18 staff.
   They will travel on minibuses and coaches.
   Each minibus costs £40 and can take up to 16 people, including at least 2 staff.
   Each coach costs £160 and can take up to 48 people, including at least 3 staff.
   What is the cheapest way of arranging transport? How much will it cost altogether?